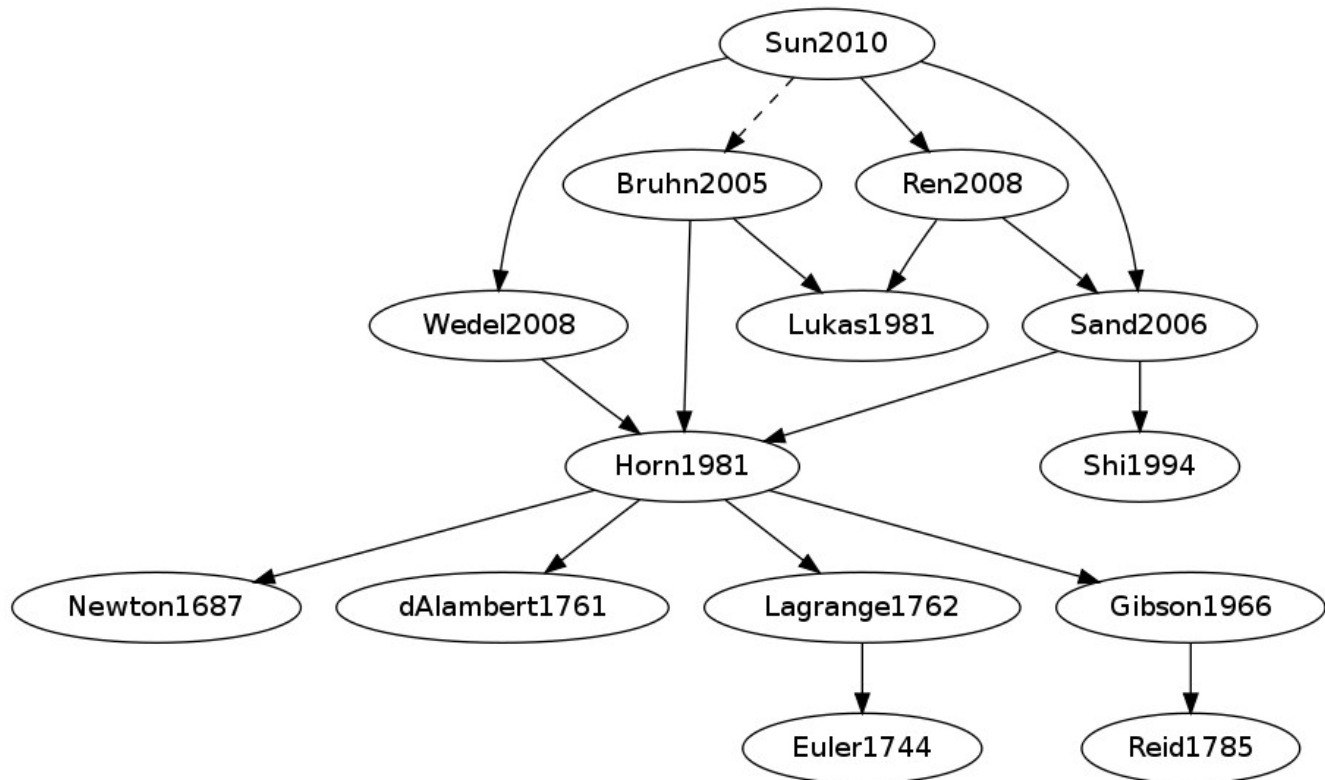


## Back to Roots: Tracking the History of Optical Flow Problem



This report is structured as follows. One can see from the scheme that there are two clusters of papers: before Horn-Shunk and after them. So we describe them consequentially.

### Novel research

*Spatial weighting.* In the final variant of their system Sun et al. use spatial weighting scheme in the way suggested by Ren. Ren has based on Lukas-Kanade method. The idea of spatial weighting could be traced further to Sand2006, who have combined a classic Horn-Shunk method with a *feature tracking* approach introduced by Shi and Tomasi.

*Median filtering.* Sun2010 refer to median filtering of intermediate flow as a critical component of their system. Moreover, they introduced a theoretical reasoning behind the median filtering. The idea was borrowed from Wedel2008 who had developed a global method based on Horn-Shunk approach.

Sun et al. also tried to implement the ideas from Bruhn2005 (*Charbonier penalty function and 7-point approximation of derivative*), but it yielded insignificant improvement, that's why the corresponding line on the scheme is dashed. But the work by Bruhn et al. is interesting itself since they combine local and global approaches in a single framework.

### Mathematical foundations

Maybe I have understood the phrase about everything newer than Aristotle will count too literally, but the results are presented. Actually, Horn and Shunk use a lot of mathematics. They tend to refer to the classical textbook, we tried to find the roots.

First, they use the finite differences to approximate derivatives. Sir Isaac Newton used this representation too (although Horn and Shunk used 5-point stencil). Although they use Laplace equation to fill the non-textured regions, which was first formulated and solved by d'Alambert. Another important branch is variation calculus. The problem of optimization of an integral in most general sense was first solved by Lagrange though it was posed by Euler. It is important to notice that modern methods use combinatorial optimization, not variation calculus. Also Horn and Shunk used Gauss-Seidel method for solving linear system, which is not too old in comparison to variation calculus.

We cannot ignore the psychological background behind the Horn and Shunk work. They refer to J.J.Gibson who formed the modern view to perception. But in the beginning of his book he refers to the work of Thomas Reid who separated sense from perception, which became an important scientific basis behind computer vision. :)

## References

- Bruhn A, Weickert J, Schnörr C. 2005. Lucas/Kanade meets Horn/Schunck: Combining local and global optic flow methods. *International Journal of Computer Vision*. 2005;61(3):211-231. Available at: <http://www.iro.umontreal.ca/~roys/ift6145/bruhn-ijcv05c.pdf>.
- Horn BK, Schunck BG. 1981. Determining optical flow. *Artificial Intelligence*. 1981;17(1-3):185-203. Available at: [http://www.umiacs.umd.edu/~pturaga/ENEE731/papers/OpticalFlow/Optical\\_Flow\\_Horn.pdf](http://www.umiacs.umd.edu/~pturaga/ENEE731/papers/OpticalFlow/Optical_Flow_Horn.pdf).
- Lucas BD, Kanade T. 1981. An iterative image registration technique with an application to stereo vision. In: *International Joint Conference on Artificial Intelligence*. Vol 130.; 121-130. Available at: <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.49.2019&rep=rep1&type=pdf>.
- Ren X. 2008. Local grouping for optical flow. In: *IEEE Conference on Computer Vision and Pattern Recognition*. Available at: <http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Local+Grouping+for+Optical+Flow#0>.
- Sand P, Teller S. 2006. Particle video: Long-range motion estimation using point trajectories. *International Journal of Computer Vision*. 2195-2202. Available at: <http://www.csail.mit.edu/~seth/pubs/SandTellerCVPR2006.pdf>.
- Sun D, Roth S, Darmstadt T, Black M. 2010. Secrets of Optical Flow Estimation and Their Principles. In: *IEEE Conference on Computer Vision and Pattern Recognition*. San-Francisco, CA. Available at: <http://www.gris.tu-darmstadt.de/~sroth/pubs/cvpr10sun.pdf>.
- Wedel A, Pock T, Zach C, Bischof H, Cremers D. 2008. An improved algorithm for TV-L1 optical flow. In: *Dagstuhl Motion Workshop*.; 1-23. Available at: <http://cvpr.in.tum.de/pub/pub/DagstuhlOpticalFlowChapter.pdf>.
- Newton I., 1687: *Philosophiæ Naturalis Principia Mathematica*.
- Lagrange J.L., 1762 "Essai d'une nouvelle méthode pour déterminer les maxima et les minima des formules intégrales indéfinies." *Miscellanea Taurinesia*.
- Euler L., 1744. "Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, sive solutio problematis isoperimetrici lattissimo sensu accepti" / "Method for finding curves having a minimal or maximal property or solutions to isoperimetric problems in the broadest accepted sense".
- von Seidel, P.L. 1874. Über ein Verfahren, die Gleichungen, auf welche die Methode der kleinsten Quadrate führt, sowie lineare Gleichungen überhaupt, durch successive Annäherung aufzulösen. *Akad.*

Wiss., Munich, mat.-nat. Abt., Abhandlungen, v. 11, no. 3, 1874, p. 81-108.

d' Alembert, J.L.R. 1761. Opuscles mathématiques. David, Paris